A new heuristic for solving the cyclic bandwidth problem

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1 Introduction

Let $G(V, E)$ be a finite undirected graph of order n and C_n a cycle graph with vertex set $|V'| = n$ and edge set E'. Given an injection $\varphi : V \to V'$ that represents an embedding of G in C_n, the cyclic bandwidth (the cost) of φ for G is given by :

$$f(G, \varphi) = \max_{\{u,v\} \in E} \{|\varphi(u) - \varphi(v)|_n\}$$

where $|x|_n = min\{|x|, n - |x|\}$ (with $1 \leq |x| \leq n - 1$) is called the *cyclic distance*, and $\varphi(u)$ denotes the label associated to vertex u. Then the Cyclic Bandwidth Problem (CBP) involves finding an embedding φ^* with minimum cyclic bandwidth, i.e.,

$$\varphi^* = \arg\min_{\varphi \in \Omega} \{f(G, \varphi)\}$$

where Ω is the set of all possible embeddings of the given graph. The embedding φ^* satisfying this condition is called an optimal embedding. CBP is known to be NP-Hard and has various applications like ring network design, VLSI designs, data structure representations, code design and interconnection networks for parallel computer systems.

2 The proposed local search heuristic for solving CBP

We propose a new heuristic algorithm for solving CBP that is based on the idea of Breakout Local Search [2, 3], which is itself a variant of Iterated Local Search. The algorithm iteratively alternates between a descent search phase (to find local optima) and a dedicated perturbation phase (to discover new promising regions). Starting from an initial solution, the algorithm improves the solution until it reaches a local optimum (intensification phase). Upon the discovery of a local optimum, the algorithm enters an adaptive diversification phase, which selectively performs a weak perturbation or a strong perturbation procedure. From the algorithmic point of view, the proposed algorithm relies on three key components : the neighborhood, the evaluation function and two types of perturbation.

Given a candidate solution (i.e., an embedding) φ in Ω represented by a permutation of the integers $\{1, 2, ..., n\}$, its neighborhood $N(\varphi)$ is the set of solutions such that $\varphi' \in N(\varphi)$ can be obtained by swapping two labels in φ. To assess the quality of a candidate solution φ, we use a refined evaluation function that calculates the cyclic bandwidth $f(G, \varphi)$ extended with a term $e(G, \varphi)$, the latter being used to discriminate solutions with the same cyclic bandwidth.

For the purpose of search intensification, the best-improvement descent strategy is applied based on the above neighborhood and refined evaluation function. For the purpose of search diversification, the algorithm triggers at first a "weak perturbation" that is ensured by directed swap or random swap moves according to a probability. As such, during the next α iterations, a best embedding or random embedding in the neighborhood is accepted and the performed moves are recorded in a tabu list to avoid reverse moves during the perturbation. In case the search fails to escape the local optimum after these α moves, the number of "weak perturbation" moves is extended to β iterations ($\beta > \alpha$, both β and α are tuned empirically). If the search is still trapped in local optimum after performing the "weak perturbation" procedure, the algorithm switches to the "strong perturbation" that applies an "$insert - shift$" move.

3 Computational results and conclusion

The proposed algorithm is tested on a popular test-suite composed of 113 benchmark graphs with a number of vertices ranging from 9 to 8192. It is worth noting that most of these graphs have special structures and their optimal cyclic bandwidths can be calculated analytically. To assess the relative performance of our algorithm, we compare our results with two best CBP methods reported in [4], i.e., a Simulated Annealing (SA_{CB}) algorithm and a Tabu Search algorithm (TS_{CB}). Due to limitation of space, we only show the results of 36 sample instances in Table 1. CB^*, SA_{CB}, TS_{CB} and "This work" represent respectively the known optimal values, the best results of SA_{CB}, the best results of TS_{CB} and the best results of our algorithm. In summary, the experiments show that our algorithm improves the best results reported in [4] for 20 instances and matches the best results for other 54 instances. However, for 39 instances, our results are worse than the current best computational results, indicating that there is still room for further improving the algorithm presented here.

TAB. 1 – Comparative results between the proposed algorithm and the reference algorithms

Instance	CB^*	SA_{CB}	TS_{CB}	This work	Instance	BC^*	SA_{CB}	TS_{CB}	This work
tree2x9	57	306	63	57	mesh3D10x10x10	80	427	252	251
tree2x7	19	52	20	19	mesh2D8x25	8	52	8	9
path475	1	115	5	7	mesh2D7x25	7	45	7	8
path300	1	63	3	4	mesh2D5x25	5	28	5	6
path200	1	40	2	3	mesh2D25x26	25	233	164	163
path175	1	31	2	3	mesh2D19x25	19	154	119	21
path150	1	28	2	3	L-bcspwr03	?	14	11	10
path125	1	21	1	3	K-dwt___234	?	20	12	11
path100	1	15	1	2	hypercube11	526	1021	570	550
P-can___445	?	149	47	46	cycle475	1	113	5	6
O-impcol_d	?	58	38	36	cycle300	1	64	3	4
mesh3D9x9x9	65	306	184	183	cycle175	1	32	1	3
mesh3D8x8x8	52	200	129	52	cycle125	1	22	1	3
mesh3D7x7x7	40	118	42	40	cycle100	1	16	1	3
mesh3D6x6x6	30	62	31	30	caterpillar44	37	321	39	38
mesh3D4x4x4	14	19	16	15	caterpillar39	33	241	34	33
mesh3D12x12x12	114	772	435	437	caterpillar35	29	182	31	29
mesh3D11x11x11	96	582	336	334	C-bcspwr01	4	5	5	4

Références

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